Reducing Uncertainty in Small-n Analysis: A Conversation

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INTRODUCTION

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An interesting dialogue has sprung up in the recent issues of *International Studies Quarterly* about the merits of certain tools of data analysis for small-n comparative case studies. Drozdova and Gaubatz began the discussion in *ISQ*'s September 2014 issue by suggesting the incorporation of some information theory quantitative methods to bolster comparative case study findings. Bear F. Braumoeller responded in the December issue by arguing that the balance between within-sample and out-of-sample inference is ultimately untenable.

They continue the conversation here at *ISQ Online*.

Response to B. Braumoeller Critique of "Reducing Uncertainty"

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We have benefited greatly from Professor Braumoeller's insightful critique of our approach to using information theory for the systematic assessment of structured, focused case studies (Braumoeller 2014, Drozdova and Gaubatz 2014). It pushed us to more clearly set out the principles and benefits of information theory for enhancing small-n analytics.

There remain, of course, important areas of disagreement. First, we'll address the concept of uncertainty, and then we'll turn to a few of the conceptual and technical issues raised by the search for appropriate metrics for making case study more systematic.

We are interested in how informative knowledge of an observed factor is about an outcome. We draw on a well-established body of work in information theory to propose the use of uncertainty reduction metrics for assessing the results of comparative case studies. Information theory is based on and represents a branch of statistics – that being a study of properties of random variables (Shannon 1948; Hogg and Craig 1995). It is, as E.T. Jaynes argues, "a type of statistical inference" which produces "the least biased estimate possible on the given information" (1957: 620).

Mutual information is a comprehensive measure suitable for identifying probabilistic relationships among variables with the complex or unknown underlying distributions that are likely in small-n work. It is a more sensitive and accurate measure of interdependency among variables because it can uncover relationships not detected by measures based around a central tendency or other limited characteristics, such as correlation, variance, etc.

Professor Braumoeller argues that policy makers are concerned with probability rather than uncertainty. Our goal is to identify explanatory factors that provide information *about* that probability and reduce the uncertainty. If the value of X is informative, it will help us know whether a successful outcome is more or less likely. Having such information may improve a policy maker's judgment under uncertainty.

The binary information metric is symmetric: we increase our knowledge of the outcome the same amount when we know either that X=1 or that X=0. This is an appropriate characteristic for correspondence measures for binary variables. The analyst will have to note which direction the relationship goes. For Professor Braumoeller's hypothetical example, $p(Y=1) = .8$ and $p(Y|X) = .2$, the uncertainty about Y is .72 if we know nothing of X. *Ex post*, if we are certain that X=1, then $p(Y=1) = .2$, and uncertainty remains .72. But it is trivial to see that Y is now much less likely. *Ex ante*, the probability of X matters for whether it can be informative about the probability of Y. The less X varies, the less informative it can be. The mutual information value of X runs between 0 if X never occurs and .54 when there is a .25 probability that X=1, which is the maximum possible value for p(X) in this scenario.

Beyond the fundamental notion of uncertainty, Professor Braumoeller's concerns highlight three essential questions about small-n analysis generally and our method in particular:

1. Does small-n analysis have inferential value?

2. If so, can quantification assist in small-n analysis, and particularly in small-n inference?

3. If so, what is the appropriate metric to use?

The first question connects to a long-standing and sometimes overly-spirited discussion. Our answer is yes, but this is not the place to reopen that debate.

Our answer to the second question is also yes, but, of course, some care and humility is required.

As we show, drawing on three prominent examples of structured, focused case studies, there has been a lack of systematic assessment in the presentation of cases. In two of the examples, tables of results were presented without any analytic overview. In the third case, no summary table was even provided. We have reviewed case studies across a number of fields, and can report that apart from those studies disciplined by Qualitative Case Analysis (QCA) methods, this lack of a comprehensive overview is exceedingly common.

Can information-theoretic tools provide direct inferences from samples to populations? Here, we emphasize that this methodology, like any other imaginable small-n approach, is not magic. With only a handful of cases, case selection issues are always going to be important. At the risk of falling back into the large-N/small-n debate, our argument is that quantifying results can maintain the nuance of small-n analysis, while disciplining the study to clarify findings and enhance replicability.

At the end of the day, the problems of drawing inference from a small sample are preserved in quantification. Small sample inference from case studies remains a domain of what George and Bennett call "contingent generalizations" (2005).

Analysts will draw inferences from small-n studies. Our argument is that they should do so guided by more concrete metrics, rather than relying only on subjective assessment. Providing a systematic and replicable measure can help make the results of small-n studies, including those conducted with QCA, much clearer.

The final question is whether this is the right measure to use. The information metric has a number of advantages: It isn't a constructed or sample-based estimate. It does not depend on sample size for convergence. It makes no distributional assumptions. It is a precise understanding of the ability of the observed values of one variable to convey information about a second variable, and a precise measure of independence when mutual information is zero. While the shape of the information metric could be approximated by any neatly concave function, such as pY(1-pY), those would be just that, approximations.

Similarly, the techniques suggested by Professor Braumoeller as isomorphic may only be so when the assumptions about underlying distributions hold (e.g., for a linear approximation). Mutual information applies exactly when we cannot verify or rely on such assumptions, as in the world of small-n case studies about highly complex underlying phenomena. Chi-square, for example, has a nearly linear relationship with uncertainty reduction, holding aside the limitations of chi-square in the presence of low density cells (a near certainty in small-n analysis).

The information metric is attractive conceptually, and has comparative advantages that have been thoroughly explored. It is straightforward to calculate. It does require the use of logs, but everything can be done simply in a spreadsheet. The primary responsibility of the analyst is to do the counting. If there are fewer than ten cases, fingers can be used.

Again, we appreciate Professor Braumoeller's attentive interest and valuable comments. These are important issues to discuss, especially since political scientists are relatively unfamiliar with recent advances in information science. Information theory is a major analytic approach that merits our exploration. We have proposed enhancing case study analysis as a good starting point for making political scientists more aware of this important work. We expand significantly on the logic and intuitions of information theory for case study analytics in a forthcoming book (SAGE 2015).

Response to "Response to B. Braumoeller Critique of 'Reducing Uncertainty'"

Bear Braumoeller
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Professors Drozdova and Gaubatz have responded graciously and usefully to my commentary on their article on information analysis (Braumoeller 2014; Drozdova and Gaubatz 2014). I do not disagree with most of what they write. Indeed, I did not disagree too much with the original article, except in that I thought it merited a few caveats.

In response to the questions that Professors Drozdova and Gaubatz raise, I agree emphatically that small-n analysis has inferential value and that quantification can assist in small-n analysis. I never meant to imply otherwise. Rather, I intended to underscore part of their own answer: that, while quantification can assist in small-n analysis, "some care and humility is required." How much, and of what nature, was the question that I sought to raise.

To take a simple example, if I saw part of a basketball game in which LeBron James took seven shots from the floor and only made one, would I be justified in concluding that he's a poor shooter? According to the information metric described by Professors Drozdova and Gaubatz, I would: I would be fairly certain—$p(Y=1)=0.143$—that he won't make a basket on a given shot. I have no argument with that conclusion. Based on the limited evidence to which I've been exposed, that is in fact my best guess.

I would, however, hedge that conclusion pretty heavily, and this is where care and humility come in. A simple binomial test tells me that, if this is a representative sample, the 95% confidence intervals on LeBron's field goal percentage are 0.004 and 0.579. As it turns out, his career field goal percentage, 0.496, lies within those confidence intervals, so missing six out of seven shots isn't *that* unusual for him. But it's not very representative, either.

I think it's important for small-n methodologists and researchers to address the uncertainty of their conclusions, even if that uncertainty is substantial. Toward that end, it is crucial for journal editors and reviewers to cut them some slack. There is nothing magical about the asterisks that festoon our tables of statistical results. But this is *really* not the place to reopen *that* debate!

Whether or not we agree on the above, I will look forward to the authors' forthcoming Sage monograph on information theory and case study analytics. The question of what we can and cannot conclude based on limited evidence is an enduring one in a field in which interesting events can be quite rare, and the authors are to be commended for adding their intellectual firepower to a good cause.

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